<u>Mathematics and Freemasonry</u>
<u>or</u>
<u>It's All a Numbers Game</u>

Introduction

I have entitled my talk today "Mathematics and Freemasonry" or "It's All A Numbers Game". I'm not sure this qualifies as Masonic Education, but I'm hoping we can have some fun with it.
When I started the preparation of this talk, my wife said, "You should start with a funny story about mathematics."
I said, "There are no funny stories about mathematics. Only funny mathematicians."
"Well that's a start." she said.

So, there were these three mathematicians standing on the campus down at Queens. An English professor walking by stopped and asked "What are you guys looking so serious about?"
One of the mathematicians replied that they were trying to determine the height of the flag pole.
"I think I can help with that." said the English professor.
It wasn't a very big pole so he lifted it out of its stand, laid it on the ground, borrowed a measuring tape from one of the mathematicians and said "There you go. It's twenty-four feet long."
As the English professor walked away, the three mathematicians looked at each other and one said, "What a dummy! We wanted the height and he gives us the length."

So, I am going to talk about some fun parts of mathematics. And you never know, I might spark an interest somewhere.

Importance of Topic

I'd like to start, though, by pointing out that mathematics, as a topic, is important. In the second degree we are led to "contemplate the intellectual faculties". The "seven or more who hold a lodge" have an allusion to the seven liberal arts and sciences - grammar, arithmetic, astronomy, rhetoric, geometry, logic and music.

The later four - arithmetic, geometry, music and astronomy - are what Pythagoras, in about 500 BC, classified as mathematics. In fact, Pythagoras gave us the term, "mathematics", which means "that which is learned".
Through the middle ages, these seven were thought to represent the sum of human learning. The first three, grammar, rhetoric and logic, known as the trivium, made up the course of study for a Bachelor of Arts degree. The

quadrivium, arithmetic, geometry, music and astronomy, made up a Master of Arts degree.

The nature of mathematics has changed, of course, but not its importance.

- In the last century, the great British essayist, Sir James Jeans said "The Great Architect of the Universe now begins to appear as a pure mathematician."
- In the 13th century, the great Renaissance scholar, Leonardo da Vinci said "Let no one read me who is not a mathematician."
- Sixteen hundred years earlier, over his academy door, Plato had inscribed "Let no one enter who is lacking in geometry."
- And in the book of Proverbs, *Wisdom,* referring to God and creation, said " I was there when He set a compass upon the face of the deep:" And the motif of God with a pair of compasses recurs through medieval art. God, the Great Architect, God the Grand Geometrician.

<u>Number Systems</u>

Before I go too far with geometry, I'd like to talk a little bit about numbers. I have two reasons:

- First numbers are so much a part of our daily life that we tend to take them for granted. We don't realize what an amazing system we have. We don't realize how young some of its major elements are.
  For instance, our decimal notation for fractions is only about 400 years old. If that doesn't seem important, think about doing long division with Roman Numerals. The idea of negative numbers and zero were unknown to the great mathematicians of antiquity. Pythagoras is believed to have committed suicide over the concept of an irrational number.
- Secondly, people have been fascinated by numbers for millennia. They have assigned numbers all kinds of mysterious properties. We tend to do this in our ritual because it is rooted in ancient sources. Most people do not take numerology very seriously today. But we can't deny the effect it has had on art and culture.

<u>Counting Numbers</u>
The numbers we are most familiar with are the counting numbers 1, 2, 3 etc.

<u>Base 10</u>
We most commonly use a base ten number system.

- This means that we have unique names for ten numbers and then we start building names from the first ones and from multiples of ten.
- So the teens are mostly names that imply ten plus something. Fourteen is four plus ten. Twenty is two tens so twenty seven is two tens plus seven.
- We probably use the base ten because we have ten fingers. We could have a different base. Some of us do count on our toes as well.

<u>Base 5</u>

In fact, people <u>have</u> used different bases for their counting systems. I can think of two systems based on five.

- One is the farm gate system used to keep track of drinks in English pubs. For each of your first four drinks, a vertical mark is put on the board. For the fifth drink, a diagonal line is drawn across to make something looking like a farm gate and the count starts over. At the end of the evening, you may not know what the count is but the bartender certainly will.
- Roman numerals also start off as a base five. We count 1, 2, 3, 4 with vertical lines and then use a V for the number five. From six to nine, we use vertical lines plus the V and indicate ten by a cross.

<u>Base 20</u>

And, of course, there's lots of legacy today of a number system based on twenty.

- The French term for eighty is not eight tens but rather four twenties "quatre vingt". There is a hospital in Paris established in the thirteenth century which takes its name from the fact that it had three hundred beds. However, it is not called Hopital Trois Cent or three hundreds. It is called Hopital Quinze Vingt or Hospital fifteen twenties.
- The Irish term for forty as well as Danish words for forty, sixty and eighty have similar structure.

<u>The Score</u>

We have the same sort of legacies in English, of course, with the term "score".

- What American, or Canadian, for that matter, is not stirred by the opening words of Lincoln's Gettysburg address "Four score and seven years ago, our forefathers brought forth on this continent a new nation, forged in liberty and dedicated to the proposition that all men are created equal." How much impact would be lost if he had said "Eighty seven years ago a bunch of guys got together to complain about taxes. "?

- And then there is the poem from my misspent youth called "Longevity". It goes like this:

  Horses and mules live thirty years
  Without a knowledge of wines and beers;
  Goats and sheep at twenty die
  And never taste of Scotch or Rye;
  The cow drinks water by the ton
  And at eighteen is mostly done;
  The dog at fifteen cashes in
  And without the aid of rum or gin;
  The cat in milk and water soaks
  And then in twelve short years it croaks;
  The modest, sober, bone-dry hen
  Lays eggs for nogs, then dies at ten.

  All animals are strictly dry,
  They sinless live and early die.

But sinful, ginful, rum-soaked men–
Survive for three-score years and ten!
And some of them, a very few,
Stay pickled 'til they're ninety-two

Without the word "score", that poem wouldn't rhyme and it wouldn't scan!

Twelve

So much for the twenties. Where does the magic number twelve come from?

- Why do we divide the day into two twelve hour periods instead of two tens or twenty?
- And what's with the twenty four inch guage (two twelves) that we value so highly in Masonry?
- These things are usually regarded as the legacy of some culture which used a base twelve number system. They may have counted on three nuckles of the four fingers of their hand.
- Or they may have counted from one to ten on their fingers just as we do . . . but I wonder what they did for the other two digits. I'll come back to twelve later.

Sixty

And then there is the mysterious sixty. This was the counting base used by Sumarians in Mesoptamia as far back as the fourth millenium BC.

- Our legacy from this has us dividing the hour into sixty minutes and the minute into sixty seconds?
- Why would these people have chosen sixty? Perhaps because it is the first number which is divisible by 1, 2, 3, 4, 5 and 6. But a more likely explanation is the union of two cultures - one of which used base twelve and the other base 5.
- Whatever the reason, it's had a big influence on the way we do things. Having 360 degrees in a circle has defined the way we navigate the earth's surface and the way we measure distance.
- And think of the pain and confusion if, in our metrification programs, we had tried to switch to a hundred minute clock. Buying butter by the gram and gas by the litre would have been trivial by comparison.
- And suppose they had divided the circle into 400 equal parts instead of 360. The fourth part of a circle would then by 100 degrees - not 90. It's inconceivable! We'd have to change Masonic ritual.

Pythagorean system

The Junior Warden's lecture in our second degree says that "The usages and customs of Masonry have ever corresonded to those of the Egyptian philosophers". It goes on to point out parallels with "the Pythagorean system". There are similarities:

- The Pythagoreans had signs of recognition and took vows of secrecy.

- They had three degrees - novice, first degree of initiation and Mathematician.
- And they had a rigid moral code.

We know about the Pythagorean society because the secrets were eventually committed to writing. We don't know much about Pythagorus, himself. We do know that his influence was far reaching.

Pythagoras and numbers

The Pythagoreans were fascinated with numbers and drew parallels between numbers and all kinds of things.

- One of the first divisions they made was between masculine (odd) numbers and feminine (even) numbers. Even numbers were considered weaker because when you divided them in two, there was nothing in the centre (no remainder).
- It's interesting to note that the cardinal virtues of Temperance, Fortitude, Prudence and Justice represented by the tassels in the corners of the lodge have a feminine number (four) while the seven deadly sins have a masculine number, seven. You guys are bad.

Number symbolism

The number one was viewed by the Pythagoreans as the source of all numbers - essential, indivisible. It is also commonly associated with the diety. "*In the beginning God created* . . . so, in the beginning there was only one.

Two can be duality or complimentary - Adam and Eve, the sun and the moon - but also opposites - Yin and Yang, good and evil, loss of unity.

Is three a special number? Well, there are:

There are the three great lights of Masonry and
the three lesser lights.
There are the three lessons of the charity lecture and
three who rule a lodge
There are the three great pillars - Wisdom Strength and Beauty
and the three orders of architecture.

Outside of Masonry, for starters, there is
the Holy Trinity,
three wise men,
give me three good reasons and
the three little pigs.

Sounds like a pretty special number to me!

Five turns up in our ritual a few times. There are the five to hold a lodge and the five noble orders of architecture. When King Solomon sent fifteen trusty Fellowcraft out to search for the Grand Master, he divided them into three lodges of five. And of course there are the five points.

Pythagoras regarded seven as the virgin number because it has no factors and there is no known construction for dividing a circle into seven. It took seven years to build the temple and it takes seven Masons to make a lodge perfect. The rubrics associated with our ritual do not specify if the Masons have to be virgins.

But seven has always been important -

starting with the acts of creation and the days of the week,
the days of the Passover,
the seven deadly sins and
the seven virtues (Faith, Hope and Charity added to the other four).

By the way, for those of you interested, the seven deadly sins are Gluttony, Luxury, Sloth, Lechery, Wrath, Avarice andEnvy. Have fun!

Geometry

Story of the sons

Once, in Africa, there was a great king who took unto himself three wives. For one wife he made a bed of hippopotamus hide. For another he made a bed of antelop hide and for the third he made a bed of lion skin. In the course of time, all three wives became with child.

The wife on the bed made of lion skin delivered a fine healthy son. The wife who had lain on the antelop hide delivered two fine healthy sons and the wife with bed of hippopotamus hide delivered three sons.

A wise man was summoned to explain this phenomenon. After some study, the wise man, who was really just a simple country doctor, made a profound statement which was later misquoted and ascribed to the mathematician Pythagoras. He said "Lo, the sons on the hippopotamus hide are equal to the sum of the sons on the other two hides".

Brethren, that was a test. If you groaned, you're OK. If you laughed, you're a little strange but probably still OK. If you had no reaction at all, it's time to time to talk to your grandchildren about grade ten math.

Origins of Geometry

What we know as geometry had its beginnings in ancient Egypt. The word comes from *geo* or earth and of course *metry* is measurement. So geometry was, literaly, "land measurement". Land was alotted to farmers who were taxed on their production. Each year, the Nile flooded its banks and washed away key property markers and the land had to be re-measured. It was a way of ensuring full but equitable taxation.

Rope Pulling

The key instrument used for this was a knotted rope **(show it)**. In fact the term hypotenuse has its origins in the Egyptian for rope puller.

Geometry as it was practiced by the ancient Egyptians was very different from what we know as geometry today. Egyptian geometry really was measurement so there were some very interesting restrictions on what they could do. They lacked the abstraction of similar triangles so they could not calculate the height of the pyramid even though their very practical measurement techniques allowed them to build it.

Euclid was the first to build a set of logical relationships between well defined abstract entities. And the book he wrote based on this was second only to the Bible in its impact on modern culture. The great cathedrals of the middle ages were built on Euclidean Geometry. The machinery of the industrial revolution was built on Euclidean geometry. Modern bridges, road systems, skyscapers and aircraft are built using Euclidian geometry. Euclidian geometry only started to hit limits with space travel and modern physics but that needn't worry us too much.

<u>47th Proposition</u>
So what's with the 47th proposition? Well, it's the one that says "The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides". The Past Master's jewel, with which we are all familiar, shows the construction for one proof of the proposition. In 1940, mathematician Elisha Scott Loomis published a book with no less than 367 proofs, including one by Leonardo da Vinci and one by US president James Garfield.

<u>The Pythagoran Triangle</u>
So what's with Pythagorus? Well, remember that Pythagorus was not really a geometrician as we know them. He was a numbers guy. What's more, he only had counting numbers to work with. But he did come up with a bunch of things that became known as Pythagorean triangles. These are right angled triangles whose sides are whole numbers.

<u>Triangle to Circle</u>
I have here an Egyptian rope and if I could have some assistance, I'll show you the simplest Pythagorean triangle.
demo (rope with twelve equally spaced knots)
So there's the right triangle with its square corner. The sides have length 3, 4 and 5 spaces. The square of length 3 has an area of 9. The square of length 4 has an area of 16. And added together they make 25 - the area of the square on the hypotenuse.

But there's more. The length of the perimeter is 12 - 3 plus 4 plus 5. If we lay this out in a circle, we have twelve equal divisions.
So there we have the wonderful twelve again -
half a twenty four inch gauge
the signs of the zodiac,
the hours of the day,
the Mayan calendar,
the phases of the moon,

the number of apostles,
the perimeter of the simplest Pythagorean triangle,

The New Math

I'd like to conclude with a few thoughts on modern mathematics. I've talked a little bit about geometry and numbers and some of the ancient history of mathematics but it has really exploded in the last 500 years. Whole branches of mathematics have been established to characterize different physical phenomena or solve certain classes of problems. And, of course, the more knowledge grows, the more it gets classified and labeled.

So Webster's New Collegiate Dictionary defines arithmetic as "the science of positive real numbers". The Encyclopedia Britannica defines Algebra as "the generalization and extension of arithmetic" and Geometry as "the study of the properties of space". Trigonometry is "the study of the properties of triangles and their application".

Trigonometry
The best known practitioners of trigonometry are land surveyors. The secret of their entire craft lies in the fact that a triangle has three sides and three angles and if you know the length of one side and a couple of angles or two sides and one of the angles, you can calculate the other three.

So surveyors have worked their way back and forth across the country laying out triangles. The problems they have solved range from determining where your neighbor's fence should be to calculation of the distance between mountain tops and across great bodies of water.

Of course, simple triangles really only work on a flat surface so for long distances, allowances have to be made for the curvature of the earth. This has led to yet another branch of mathematics called spherical trigonometry or trigonometry on the surface of a sphere.

Here's an example to illustrate the problem of working on a sphere. Suppose you stand in the middle of a lodge facing south. You will be looking at the Junior Warden. Now if you turn 90 degrees to the right, you will be facing the Senior Warden in the West. Turn another 90 degrees and you will face the Chaplin in the North and yet another 90 will have you looking at the Worshipful Master in the East.

Now imagine the lodge centered on the North Pole. If you face the Junior Warden, you will be looking south as you would expect. If you turn 90 degrees to the right, you will be facing the Senior Warden as you would expect but he is now in the South. Another 90 degrees will have you facing the Chaplin, also in the South and another 90 degrees brings you to the Worshipful Master . . . in the South. If you're standing at the North pole, south is the only way to look.

Calculus

Every school child has a mental picture of Isaac Newton sitting under an apple tree on the campus at Cambridge University and getting hit on the head by a falling apple. We have been led to believe that this brought him to the discovery of gravity. I expect, though, that people already knew that when things fell, they fell down. In fact, several centuries later, the same thing happened to a student at Berkley University in California and he proclaimed "Gravity is a myth. The earth sucks."

While Newton did not "discover" gravity, he was the first to recognize its universal nature. What he recognized was that every body exerts a gravitational pull on every other body. The force of the attraction between two bodies is dependent on the mass of the bodies and the distance between them. So, on the moon, you would feel that you weighed only one sixth what you do on earth because the mass of the moon is so much less than that of the earth. Newton used the universal law of gravity to explain the motion of planets and to establish a theory of physics which was not seriously challenged until Einstein's theory of relativity in the 20th century.

"So what!" you ask. Well, in order to do this, Newton had to develop his own system of mathematics. Building on concepts dating back to Archimedes in the 3rd century BC and Indian mathematicians several centuries earlier, Newton and a German mathematician named Leibnitz developed what is known today as calculus. From its initial application in mechanics and planetary motion, calculus went on to develop equations of electromagnetic theory, heat transfer and fluid dynamics. Calculus is now used in nearly every branch of science.

Nearly every scientific break-through has been accompanied by advances in mathematics. These were required to characterize the new discoveries and to articulate advancing hypotheses.

Operational Research

But it wasn't until the second word war that we started to see mathematics used extensively outside the physical sciences. A number of academics were conscripted into the British war effort to address problems in logistics. They established a discipline known as Operational Research or OR. The Americans followed suit and both groups had notable success in a number of areas.

One of the early American successes had to do with serving efficiency in defense establishment cafeterias. After what I'm sure was heavy application of queuing theory, someone came up with the idea of posting a chart in the cafeteria showing the length of the line-up at intervals between 11:30 AM and 1:30 PM. Serving became more efficient simply because people avoided the rush.

Some of the most successful applications of OR had to do with force deployment. Britain, early on, realized that shipping across the Atlantic was best done by convoy but how big should the convoys be? Convoy speed was limited by its slowest member so big convoys had more time exposed to potential U-boat attack. On the other hand, having a lot of small convoys spread the defenses very thin. The mathematicians concluded that the size of the convoy did not matter. The critical factor for defense was the absolute number of defensive vessels.

One writer has claimed that this numbers issue was the deciding factor in the battle of the Atlantic. In November and December of 1942, convoys under Canadian escort suffered such heavy losses that the force was pulled and sent for more training. A British escort in January of 1943 fared no better, nor did an American escort in February. Two convoys in March lost 23 ships and failed to sink a single U-boat. In May of 1943, the allies sank 40 U-boats and the battle was over. There had been advances in ant-submarine technology through this period but the single biggest factor in May was concentration of defensive forces - purely an issue of numbers.

Some of the mathematicians' recommendations were quite counter-intuitive. In the Pacific campaign, the American air force doubled the bombing accuracy of B-29s by decreasing their mission time. They increased in-flight training time from 4% to 10% and decreased mission time from 96% to 90% with really significant pay-off.

The British air force did an extensive survey of damage to bombers returning from continental raids and recommended additional armor on parts of the plane that had suffered most damage. The mathematicians recommended armor plating the areas which had <u>not</u> been damaged. They argued, successfully, that the survey was biased because the air force had only looked at the planes that had returned. They didn't see the hits that brought the others down.

The techniques used by Operations Research people during WWII - queuing theory, statistics, mathematical modeling, simulation - have become part of the standard management tool kit. Many are supported by commercially available computer software. Statistical charts appear in the newspapers and military effectiveness measures such as "over-kill" have made it into our common language.

<u>Pitfalls</u>
Of course, this common use of quantification and mathematical language just gives us more things to be wary of. I'm sure you're all familiar with the old line "Figures never lie but liars sure can figure." And, while the figures may not lie, they can suck us into lying to ourselves.

One thing to watch out for is what I call "the illusion of accuracy".

It's illustrated by the story of the young man working at the Royal Tyrrell Museum in Drumheller. One day a visitor asked him: "Can you tell me how old the skeleton of that T-Rex is?"

"It is precisely 60 million and three years, two months, and eighteen days old."
"How can you know that with such precision?" the visitor asked.
"Well, when I started working here, one of the scientists told me that the skeleton was 60 million years old - and that was precisely three years, two months, and eighteen days ago..."

A commonly held business belief, today, is that you can't manage what you can't measure. There is probably an element of truth to this but just because you are measuring does not mean you are managing and measuring to great precision certainly doesn't mean you are managing well.

Statistics

I'm sure that everyone is familiar with the Benjamin Disraeli quote that there are three kinds of lies - lies, damn lies and statistics. You may be less familiar with the definition of a statistician as "a person who can draw a mathematically precise line from an unwarranted assumption to a foregone conclusion".

Once when Stats Canada was hiring mathematicians, three recent graduates were invited for an interview. One had a degree in pure mathematics, another one in applied math, and the third had a B.Sc. in statistics.
All three were asked the same question: "What is one third plus two thirds?"
The pure mathematician said "It's one."
The applied mathematician took out his pocket calculator, punched in the numbers, and replies, "It's 0.999999999."
When the interviewer turned to the third candidate, the statistician asked, "What do you want it to be?"

Common errors in day-to-day uses of statistics include misleading graphical presentations, poor definitions and what I call misplaced baseline.

The misplaced baseline is frequently used to give a dramatic headline to a news story. The headline says that some particular course of action will reduce your chances of some obscure disease by 30%. They don't bother to tell you that, unless you are a Patagonian dwarf, your chances of getting that disease are about one in ten million. So a 30% reduction is hardly worth changing your life style.

Sometimes these stories can have a very serious affect. A couple of years ago, there were big headlines about a statistical relationship between estrogen treatments and breast cancer. Because the baseline was unclear and the statistics poorly understood, a lot of women stopped the treatments when they shouldn't have.

Poor definitions frequently come from a poor understanding of statistics. On a recent radio interview, a person concerned with obesity in children was asked to define obesity. The reply was "We say a person is obese if their body weight index is above the 95th percentile for their age and height." By defining obesity in this way, you guarantee that 5% of the population will be obese and 95% not obese.

That's what "95th percentile" means. To have meaning, obesity must be defined in terms of something independent of the population sample.

Pareto Principle

A pseudo statistical tool that's becoming more and more common is the 80-20 rule. Quality control guru, Joseph Juran, labeled this the Pareto Principle after an Italian economist who noted in the 19th century that 80% of the wealth was held by 20% of the people. It was later noted that there is a similar "clumping" of relationships to be seen in many places.

The Wikipedia entry on Pareto points out that we wear 20% of our wardrobe 80% of the time and we spend 80% of our time with 20% of our acquaintances. This clumping can be extremely useful for focusing our attention and is the basis of important tools in quality control, logistics and procurement. The problem lies in trying to make it too mathematical.

For starters, there is nothing sacred about 80-20. It could as easily be 65-35 or 90-10. In fact, the rules of 0-0 and 100-100 always apply. To go back to Pareto's data, 0% of the wealth will always be held by 0% of the population and 100% of the wealth will always be held by 100% of the population. What's more, there is no need for the two numbers to add up to 100 since they describe percentages of completely different things. This is very disconcerting for people who sit in management meetings and check to see if the percentages add up to 100 - and you all know some like that! And finally, Pareto is very open to the concern I raised earlier about misleading precision. This is a course tool for setting priorities, not a precise instrument.

Incidentally, the 80-20 rule is also known as "the law of the vital few" and "the principal of factor sparsity" . . . Who says mathematicians can't be funny?

Conclusion

I seem to have drifted a long way from Pythagorus but I hope some of this has value for you. Maybe it will inspire a healthy cynicism the next time an ambitious reporter write a sensational story about 5% of gas pumps being off by a couple of ounces per fill-up. Or maybe it will challenge you to read the numbers in the reports of the Grand Lodge communications instead of going straight to Grand Historian's poetry selection. Or maybe it will inspire an appreciation for the way some of these things creep into real life.

A friend of my father's who had no formal mathematical training used to think that statistics were absolutely wonderful.
"How so?" I asked him once.
"Well, according to statistics, there are 42 million alligator eggs laid every year. Of those, only about half get hatched. Of those that hatch, three fourths are eaten by predators in the first 36 days. Of the rest, only 5 percent get to be a year old for one reason or another. Isn't statistics wonderful?"

"What's so wonderful about all that?" I asked.
He said, "If it weren't for statistics, we'd be up to our asses in alligators!"
Thank you, Brethren. It's been a great pleasure being with you today.

Thank you, Brethren. It's been a pleasure visiting here today.

J. William Bowick,
Templum Fidelis Lodge
September, 2008